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Mach Wave Emission from a High-Temperature Supersonic Jet

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The paper considers the compressible Rayleigh equation as a model for the Mach wave emission mechanism associated with high-temperature supersonic jets. Solutions to the compressible Rayleigh equation reveal the existence of several families of supersonically convecting instability waves. These waves directly radiate noise to the jet far field. The predicted noise characteristics are compared to previously acquired experimental data for an axisymmetric Mach 2 fully pressure balanced jet (i.e., $P_e/P_a = 1.0$) operating over a range of jet total temperatures from ambient to 1370 K. The results of this comparison show that the first-order supersonic instability wave and the Kelvin-Helmholtz first-, second-, and third-order modes have directional radiation characteristics that are in agreement with observed data. The assumption of equal initial amplitudes for all of the waves leads to the conclusion that the flapping mode of instability dominates the noise radiation process of supersonic jets. At a jet temperature of 1370 K, supersonic instability waves are predicted to dominate the noise radiated at high frequency at narrow angles to the jet axis.

Nomenclature

$A(x)$	= fluctuating pressure amplitude
b	= half-width of shear layer
c_a	= ambient sound speed
D	= jet exit diameter
f	= frequency, Hz
$G(x_n)$	= total growth integral, Eq. (3)
k	= axial wave number
k_c	= critical axial wave number, $\rho_a^{\frac{1}{2}} M_j \omega$
M_j	= fully expanded jet Mach number
n, m	= azimuthal and radial mode numbers
P_e, P_a	= jet exit static and ambient pressure
$p(r, \phi, x)$	= fluctuating jet pressure
$\hat{p}(r)$	= fluctuating pressure eigenfunction
r, R_j	= radial jet coordinate and jet nozzle radius
St	= Strouhal number, fD/V_j
T_0	= jet supply total temperature
$\bar{u}(r, \phi, x)$	= mean axial jet velocity
V_j	= fully expanded jet velocity
x	= axial jet coordinate
x_n	= axial location of neutral point
α	= axial wave number
α_r, α_i	= real and imaginary part of axial wave number
θ	= angle of sound emission to jet axis
ν_t	= eddy kinematic viscosity
ρ_a	= ambient density
$\bar{\rho}(r, \phi, x)$	= mean jet density
ϕ	= azimuthal jet coordinate
Ω	= convective frequency, $\omega - \alpha\bar{u}$
ω	= instability wave frequency (real), rad.

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I. Introduction

NOISE generated by a fully pressure balanced supersonic jet is dominated by ballistic-like sources. These noise sources originate from the convection of turbulence at speeds that exceed ambient sound speed. Phillips[1] first described this mechanism through the development of a convected wave equation. Later Ffowcs Williams[2] developed theoretical predictions for the amplitude and directional characteristics of the mechanism which agreed with some of the observed features of rocket engine data. The approach taken by Phillips and Ffowcs Williams is based on the Lighthill acoustic analogy approach. This approach is computationally complex, requiring knowledge of the second derivative of two point space time correlations of the Lighthill stress tensor.

It has been well established that coherent large-scale structures play a dominant role in the noise generation process of supersonic jets. Experimental investigations by Troutt and McLaughlin,[3] for low to moderate Reynolds number unheated round jets, and by Seiner et al.[4] and Lepicovsky et al.,[5] for high Reynolds number jets, have shown that these coherent turbulent structures are important to supersonic jet noise generation and that these structures can be considered as a superposition of instability waves. Instability wave theory, which treats the large-scale turbulent structure as a linear superposition of instability waves, has been used by Tam and Burton[6] to model the noise sources produced by cold round jets. This model theory is much simpler to implement and experimentally verify than is the analogy approach. Instability theory, as this paper also shows, is particularly relevant to prediction of important features associated with the Mach wave emission mechanism.

More recently, Tam and Hu[7] have obtained solutions to the compressible Rayleigh equation that indicate the existence of three families of waves for high-temperature jet plumes. Based on their analysis, predicted convection velocities agreed with the high Mach number experimental shock tube driven free jet results of Oertel.[8] Of the three families of waves, two were found to convect supersonically relative to the ambient sound speed. These two wave families were designated as Kelvin-Helmholtz (K-H) and supersonic instability waves (SIW). Both the K-H and SIW wave families were expected to be capable of producing intense noise radiation. The third family of waves had a subsonic convection velocity in the jet temperature range considered and were thus not expected to produce noise radiation.

Tam et al.[9] compared the results of linear instability wave theory to the round hot jet data of Tanna et al.[10] Good agreement was found between the Strouhal number for maximum sound radiation and the most amplified instability wave. Since the data set was limited to 730K, little comparison could be made to acoustic properties associated with the SIW. However, the recent spark schlieren results of Seiner et al.[11] demonstrated that the predicted wave angle for both the K-H and SIW waves agreed well with the intense wave system of high-temperature supersonic jets. These experiments were conducted using a Mach 2 round convergent-divergent nozzle with jet total temperatures ranging from ambient to 1370 K.

The purpose of this paper is to extend the previous analysis of Tam et al.[9] to this new set of hot jet data. To accomplish this, solutions to the compressible Rayleigh equation are obtained for each of the four Strouhal numbers reported by Seiner et al.[11] for the jet total temperatures of 313, 755, 1114, and 1370 K. This data is associated with a Mach 2 convergent-divergent round nozzle operating fully pressure balanced (i.e., $P_e/P_a = 1.0$) with a shock free plume. Both the wave angle and normalized peak noise amplitude level for these temperatures and Strouhal numbers are tabulated. The total growth integral for the most fundamental K-H and SIW waves is presented to demonstrate the Strouhal number range of importance for the various modes and chart their dependence on jet total temperature. Since the radiated noise characteristics also depend on the mode's phase speed, axial growth rate, and wave number spectrum, these results are presented for the 1370K jet total temperature. Finally direct comparison is made between predicted far-field noise and experimental data.

II. Computational Procedure

Linear instability wave theory for supersonic jets is now well known. It can be shown that the development of an instability wave of fixed real frequency ω is governed by the compressible Rayleigh equation,

$$\frac{\partial^2 \hat{p}}{\partial r^2} + \frac{\partial \hat{p}}{\partial r}\left[\frac{2\alpha}{\Omega}\frac{\partial \bar{u}}{\partial r} + \frac{1}{r} - \frac{1}{\bar{\rho}}\frac{\partial \bar{\rho}}{\partial r}\right] + \left[\bar{\rho}M_j^2\Omega^2 - \frac{n^2}{r^2} - \alpha^2\right]\hat{p} = 0 \qquad (1)$$

where $\Omega = \omega - \alpha\bar{u}$. Here $\bar{u}$ and $\bar{\rho}$ are the mean velocity and density, respectively, and M_j is the fully expanded jet Mach number. In Eq. (1), n is the azimuthal mode number, and α is the axial wave number or eigenvalue of the problem. For convenience, a cylindrical polar coordinate system (r, ϕ, x) is chosen with the jet axis in the x direction. Here, it is assumed that the flow is locally parallel and that the fluctuating pressure takes the form

$$p(r, \phi, x, t) = A(x)\hat{p}(r)\exp[i(\alpha x + n\phi - \omega t)] \qquad (2)$$

where $A(x)$ is the amplitude function.

The steps involved in determining the wave number α as the wave propagates downstream and its associated noise field are described in Tam and Burton[6] and in Seiner and Bhat.[12] For conciseness, these steps will not be repeated in this paper. The analysis and the numerical calculations performed in these papers[6,12] were for jets at low temperatures. This approach can also be applied to high-temperature jets as the effects of total temperature is taken into account by the temperature ratio (jet/ambient) term in the Crocco's equation for mean density.

The procedure described in these papers can also be used to determine the axial development of the supersonic instability waves. The only difference is that in the inviscid limit, the Kelvin-Helmholtz waves can be analytically continued into the damped region (i.e., region of negative growth rate), but the supersonic instability waves cannot. As shown by Tam et al.,[9] an eddy viscosity term is required to be included to continue the calculation of the SIW into the damped region. In all of the calculations performed here, the local eddy Reynolds number $\nu_t b/V_j$ is taken equal to 500, where ν_t is the eddy kinematic viscosity and b is the half-width of the shear layer.

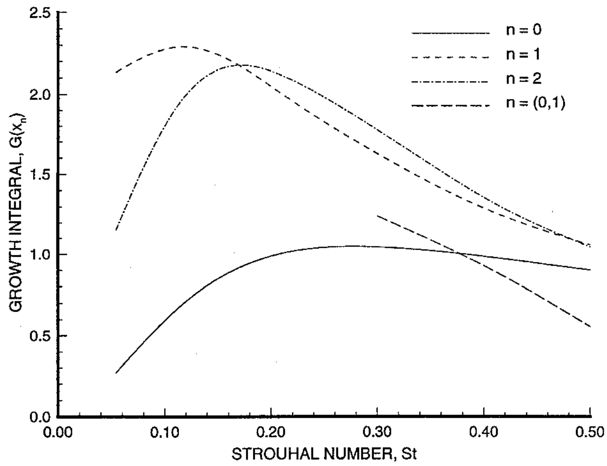

Fig. 1 Total growth integral, M_j = 2.0, T_0 = 755 K.

Instability wave theory suggests that the noise characteristics of hot supersonic jets in the peak radiation direction are related to those of the most amplified instability wave. The total amplification of an instability wave of frequency ω and mode number n is related to the growth rate of the wave, which is functionally related to the sign and magnitude of α_i, the imaginary part of the axial wave number α. The total amplification or the growth integral is given by

$$G(x_n) = \left[\int_0^{x_n} -\alpha_i \,\mathrm{d}x\right] \qquad (3)$$

where x_n is the axial location at which the wave becomes neutral, i.e., $\alpha_i = 0$. So, to determine the most amplified wave, the growth rates have to be calculated for various modes and frequencies. The wave with the highest total growth can then be determined for each jet condition under consideration. The total growth integral for the K-H instability waves (mode numbers n = 0, 1, and 2) and, where appropriate, the SIW for (n, m) = (0,1) have been calculated for a Mach number 2.0 jet. Here the index m refers to the wave's radial mode number.

The existence of supersonic instability waves have been determined using the vortex-sheet model (VSM), as previously described by Tam and Hu.[7] The definition of these waves is ambiguous as, in some cases, a wave with supersonic phase speed (using VSM) is seen to become subsonic using the eddy viscosity model. However, to avoid any further confusion, this paper will define the existence of a supersonic instability wave based on the observance of a supersonic phase speed from the VSM analysis. In all of the calculations, the mean velocity data acquired by Seiner et al.[11] has been used to determine the parameters for the mean velocity profile.

III. Numerical Results

Total Growth Integral

Figures 1 and 2 show the total growth integral $G(x_n)$, defined in Eq. (3), as a function of Strouhal number for the two jet total temperatures of 755 K and 1370 K. These results indicate that the helical $n = 1$ and 2 modes dominate over the axisymmetric $n = 0$ mode for both temperatures over the entire Strouhal number range of interest. This is especially true at low frequencies, in particular, when $St < 0.3$. For Strouhal numbers less than 0.1, the $n = 1$ helical mode is seen to become more dominant than even the $n = 2$ helical mode. It is to be noted that the present 755K results for $G(x_n)$ are in good agreement with the previous calculations of Tam et al.[9] for a Mach 2 round jet at 730 K.

Comparison of the magnitude of $G(x_n)$ in Figs. 1 and 2 shows that its value diminishes for all K-H waves with increasing jet total temperature. However, both the magnitude of $G(x_n)$ and Strouhal range increases for the SIW with increasing jet total temperature. The SIW eventually attains as much significance as all other fundamental modes considered at the high temperature of 1370 K and Strouhal numbers near 0.2 as can be seen in Fig. 2.

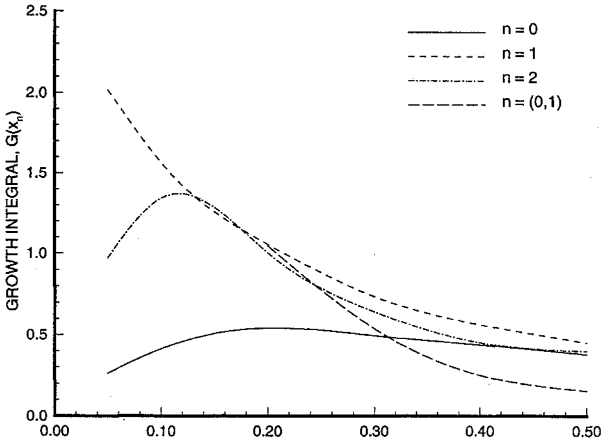

Fig. 2 Total growth integral, M_j = 2.0, T_0 = 1370 K.

Computations at the other jet total temperatures of 313 K and 1114 K reinforce the characteristics discussed earlier for the dependence of $G(x_n)$ for the various modes as a function of jet total temperature. These results suggest that axisymmetric structure plays a less significant role in generation of far-field sound and that supersonic instability waves only become important at extremely high jet temperatures.

Phase Velocity and Growth Rate

In this section, we show two important characteristics associated with the instability wave development that have a bearing on determining the far-field sound. These are the axial variation of the modal phase velocity ω/α_r and growth rate $-\alpha_i$. The angle of sound emission to the jet axis is determined from each wave's phase velocity. The axial location for peak noise emission, based on the experimental observations of Troutt and McLaughlin[3] and Seiner et al.,[4] indicate that it occurs near the axial location of each wave's neutral point. Knowing the location and emission angle for sound are important characteristics in deriving directional properties of the jet sound field.

As mentioned earlier, extensive calculations of the axial development of the instability wave and associated far-field noise have been carried out over a range of Strouhal numbers, mode numbers, and jet temperatures. To be concise, only the phase velocity and growth rate results will be shown for the 1370 K jet temperature and Strouhal numbers $St = 0.1$ and 0.4.

Figures 3 and 4 show the axial variation of the phase velocity with respect to the ambient sound speed and growth rate at the jet temperature $T_0 = 1370$ K for $St = 0.1$. The first three fundamental K-H waves are presented. For this low Strouhal number, no solution exists for the SIW. All three K-H waves have supersonic phase velocity and hence would generate intense Mach wave radiation. It is also seen that the axisymmetric mode ($n = 0$) has the highest phase speed and would therefore be expected to radiate at a higher angle to the jet axis than the helical modes ($n = 1$ and 2).

The growth rate curves of Fig. 4 indicate that the helical modes have higher growth rates and become damped much closer to the jet exit than does the axisymmetric mode. Note that the neutral point of stability for each mode is located at the axial distance x_n where the growth rate is zero.

The phase velocity and growth rate for the waves at a Strouhal number of 0.4 are shown in Figs. 5 and 6, respectively. The variations are similar to those obtained for the lower Strouhal number. However, it can be seen that the helical modes have higher phase speeds and neutral points closer to the jet exit than for those helical modes at lower frequency. This is consistent with the understanding that higher jet noise frequencies are with turbulence structure closer to the nozzle exit. The calculations for the supersonic instability wave $(n, m) = (0, 1)$ are stopped at the point where the amplitude function completes the cycle of growth and decay. Based on the total growth integral results of Fig. 2, the SIW component at $St = 0.4$ experiences a much smaller growth, as shown in Fig. 6, than its counterpart at $St = 0.2$.

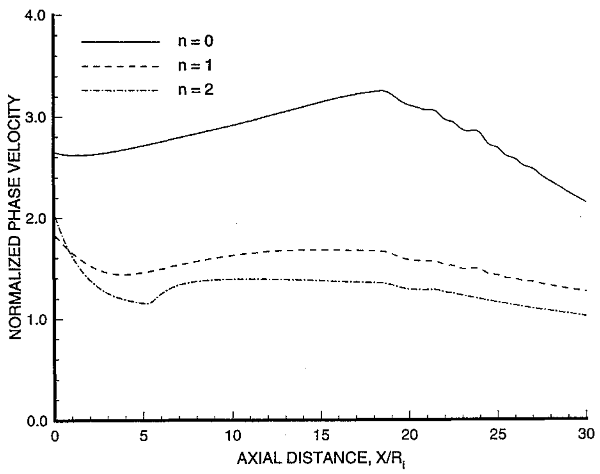

Fig. 3 Axial variation of phase velocity, M_j = 2.0, T_0 = 1370 K, St = 0.1.

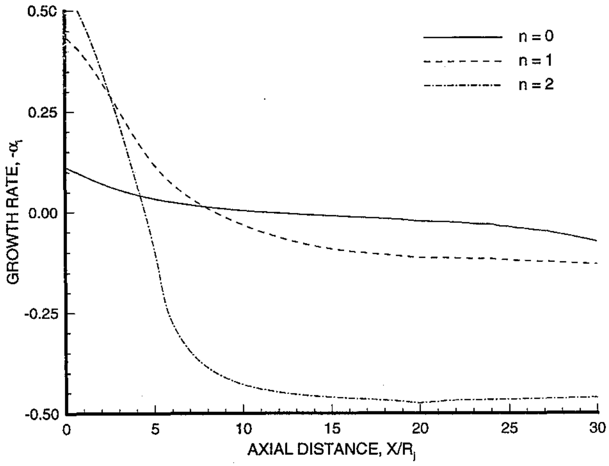

Fig. 4 Axial variation of growth rate, M_j = 2.0, T_0 = 1370 K, St = 0.1.

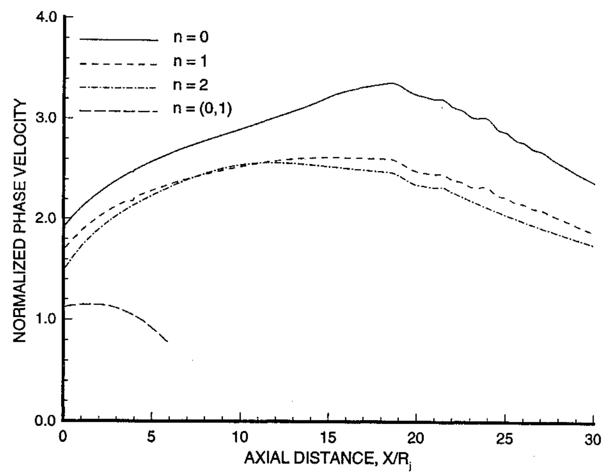

Fig. 5 Axial variation of phase velocity, M_j = 2.0, T_0 = 1370 K, St = 0.4.

Wave Number Spectra

The inverse Fourier transform of the wave number spectrum determines the amplitude function $A(x)$ of Eq. (2). In this sense, the wave number spectrum is an important barometer for the diagnosis of sound radiation from supersonic jets using the wave model approach.

The amplitude of the wave number spectra for the two Strouhal number cases considered earlier are shown in Fig. 7 for $St = 0.1$ and in Fig. 8 for $St = 0.4$. Assuming an equal initial amplitude for all of the modes, the helical mode is seen to dominate at low Strouhal

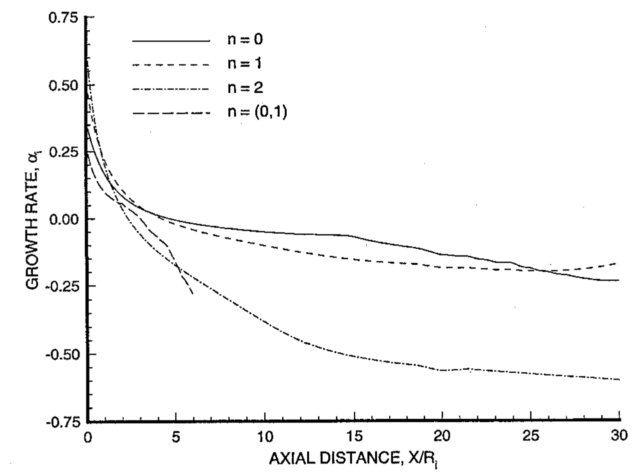

Fig. 6 Axial variation of growth rate, M_j = 2.0, T_0 = 1370 K, St = 0.4.

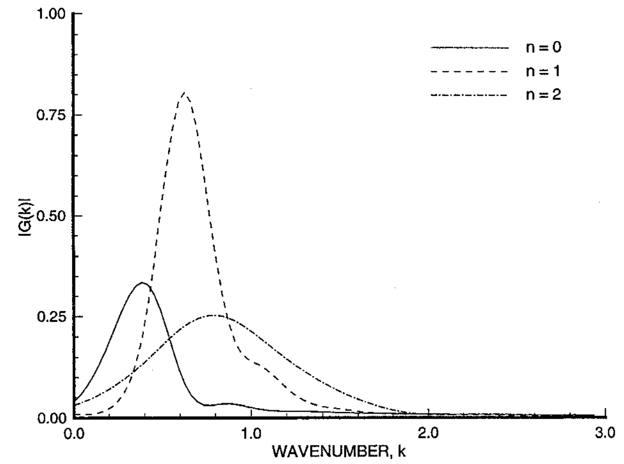

Fig. 7 Wave number spectrum, M_j = 2.0, T_0 = 1370 K, St = 0.1.

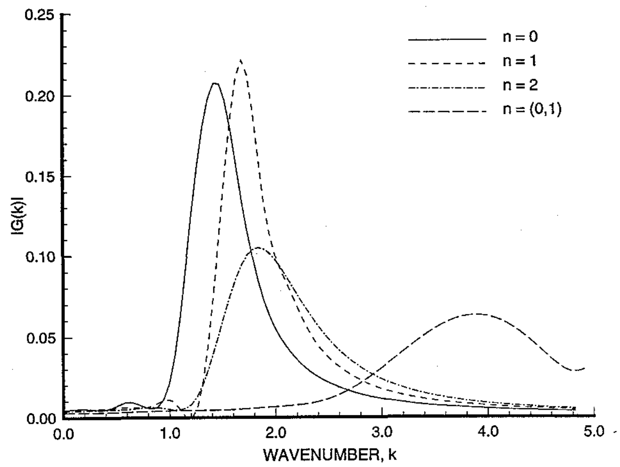

Fig. 8 Wave number spectrum, M_j = 2.0, T_0 = 1370 K, St = 0.4.

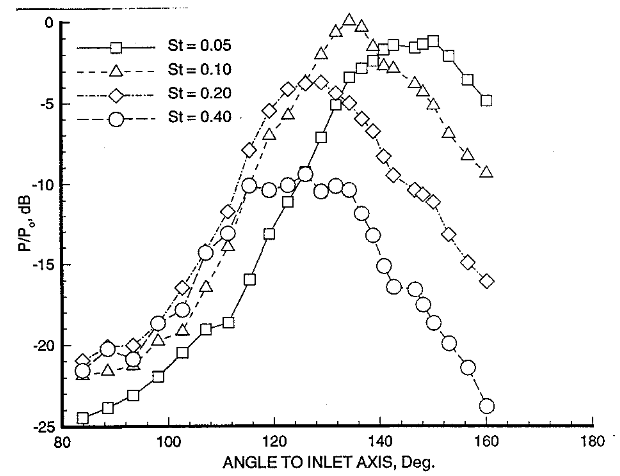

Fig. 9 Far-field directivity, M_j = 2.0, T_0 = 1370 K (experimental data).

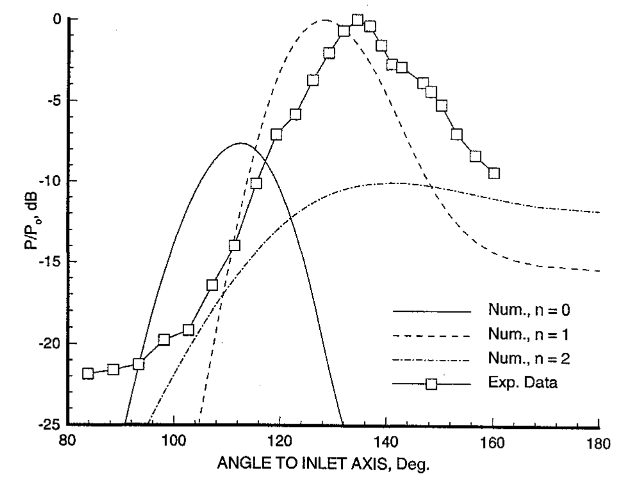

Fig. 10 Far-field directivity, M_j = 2.0, T_0 = 1370 K, St = 0.1.

number. At high Strouhal number, both axisymmetric and helical modes are essentially equivalent. However, only those wave numbers k with supersonic phase speed relative to the ambient medium would emit sound to the far field. The critical wave number is defined as $k_c = \rho_a^{1/2} M_j \omega$, where ρ_a is the ambient density. Wave numbers with smaller values than k_c will have supersonic phase speed and directly radiate sound to the far-field. The critical wave numbers for the Strouhal numbers 0.1 and 0.4 are 1.02 and 4.08, respectively. As can be seen, for both Strouhal numbers, the wave number corresponding to the peak amplitude is located well within the supersonic region, so that most of the near-field pressure disturbances would radiate into the far-field as sound. The direction of radiation of each wave number component is given by the relationship $k = k_c \cos\theta$, where θ is the angle to the jet axis in a spherical coordinate system.

IV. Predicted and Measured Far-Field Sound

Experimental far-field acoustic data from Seiner et al.[11] is shown in Fig. 9. Here the normalized far-field directivity of narrow-band spectral data (24-Hz bandwidth) for the Mach 2 convergent-divergent nozzle operating at 1370 K is presented for the Strouhal numbers $St = 0.05, 0.10, 0.20$, and 0.40 as a function of angle to the inlet axis Ψ. The data is normalized by the spectral amplitude corresponding to the maximum value P_0 among all four Strouhal number components. For this jet condition, this corresponds to $St = 0.10$. This normalization procedure is chosen since instability wave theory cannot predict absolute values for noise radiation.

Figures 10 and 11 show the predicted normalized sound pressure levels for the various modes at Strouhal numbers 0.10 and 0.40, respectively. The experimental data is also shown for convenience. In comparing the predicted and experimental data, it is important to recognize that the experimental data contains contributions from all scales of the turbulence, including those from small-scale incoherent structures. The numerical predictions only calculate the noise associated with the large-scale coherent structure. Secondly, as the initial amplitude of the various modes is not known, a more direct comparison to the experimental data by combining the contributions from the individual azimuthal modes cannot be made.

The peak radiation angle for the various modes is consistent with what can be inferred from the wave number spectra shown earlier. At low Strouhal number, it can be seen that the helical mode is dominant with an angle corresponding to the peak that best approximates the experimental data at Ψ = 136 deg. Because of the assumption of equal initial mode amplitude, it is not clear if the relative

Table 1 Angle corresponding to the peak noise level

St	Exp. Data	$n = 0$	$n = 1$	$n = 2$	$n = (0, 1)$
$T_0 = 313$ K					
0.11	156.5	134.4	—	—	—
0.19	150.1	136.1	—	—	—
0.30	146.6	139.5	151.6	153.9	—
0.40146.6	139.5	147.1	147.2	—	
$T_0 = 755$ K					
0.054	153.0	120.0	—	—	—
0.11	142.6	120.0	139.5	147.1	—
0.22	134.3	120.0	128.3	132.8	—
0.43	125.9	117.4	121.3	125.5	—
$T_0 = 1137$ K					
0.056	153.0	113.6	147.1	—	—
0.10	140.8	114.8	132.8	145.1	—
0.20	128.9	111.1	124.1	126.9	—
0.40	119.2	113.6	117.4	120.0	—
$T_0 = 1370$ K					
0.05	150.1	113.6	143.1	—	—
0.10	134.3	112.3	128.3	141.3	—
0.20	128.9	108.6	120.0	122.7	—
0.40	125.9	111.1	114.8	117.4	163.7

Table 2 Normalized peak noise level

St	Exp. Data	$n = 0$	$n = 1$	$n = 2$	$n = (0, 1)$
$T_0 = 313$ K					
0.11	0.0	−14.0	−2.7	−14.9	—
0.19	−1.1	−9.6	0.0	−10.7	—
0.30	−1.3	−8.9	−1.8	−9.5	—
0.40	−2.7	−9.9	−5.5	−11.2	—
$T_0 = 755$ K					
0.054	−0.7	−14.5	−0.7	−12.7	—
0.11	0.0	−11.9	0.0	−9.9	—
0.22	−1.6	−10.5	−3.5	−8.5	—
0.43	−6.4	−13.9	−12.1	−16.2	−28.4
$T_0 = 1137$ K					
0.056	−0.3	−12.5	0.0	−13.5	—
0.10	0.0	−12.9	−3.6	−13.8	—
0.20	−2.7	−12.4	−8.8	−15.1	—
0.40	−7.1	−15.9	−15.2	−21.4	−23.8
$T_0 = 1370$ K					
0.05	−1.2	−12.5	0.0	−13.7	—
0.10	0.0	−12.8	−5.2	−15.3	—
0.20	−3.8	−12.5	−10.7	−17.9	−28.5
0.40	−9.4	−17.0	−16.6	−22.9	−27.4

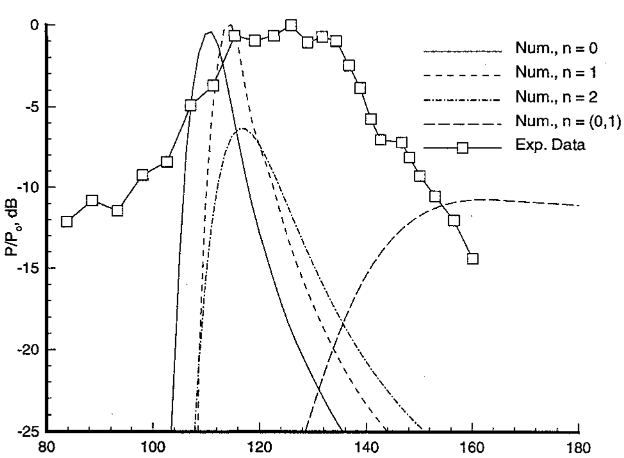

Fig. 11 Far-field directivity, $M_j = 2.0$, $T_0 = 1370$ K, $St = 0.4$.

change in amplitude between the modes can be taken as a guide in explaining the changes in slope near the peak in the $St = 0.1$ experimental data of Fig. 10. The appearance of changes in the slope of the data roughly corresponds to predicted angles of emission for the fundamental modes.

For high-frequency radiation, $St = 0.4$, the agreement between experiment and prediction is not as good as seen in Fig. 10. The axisymmetric mode is found to produce a sound level that is equivalent to that produced by the $n = 1$ helical mode. Both the second-order helical mode $n = 2$ and the SIW mode contribute much less to the overall sound. However, at angles near the jet axis, the noise is predicted to be dominated by the SIW component.

The emission angle corresponding to the predicted phase velocity for all modes, Strouhal numbers, and jet temperatures is tabulated in Table 1. The blanks in Table 1 occur when the peak of the wave number spectrum lies beyond the critical wave number k_c. Table 2 presents the normalized predicted noise levels for all four jet total temperatures considered at each of the four Strouhal numbers. Once again, in all cases considered, the initial amplitude has been assumed to be the same for the various modes. Comparing the data tabulated in Tables 1 and 2, it is clear that the first-order helical mode ($n = 1$) consistently provides the best agreement to experimental data. The agreement is seen to deteriorate at low jet temperatures and frequencies. This is because the phase speed of these instability waves relative to the ambient sound speed are subsonic, and direct Mach wave radiation is not possible. The peak emission angles for the $n = 2$ helical mode are also in good agreement with high-temperature experimental data, but the noise levels of this mode are much less than the $n = 1$ helical mode. Hence, this mode is not expected to contribute significantly to the overall noise levels. The contributions from the axisymmetric mode and supersonic instability wave are also very small for all jet conditions considered.

V. Discussion of Results

In this paper, we have shown that the compressible Rayleigh equation effectively predicts several important acoustic features of the Mach wave sound field associated with high-temperature supersonic jet plumes. Consistently, over the entire range of fully pressure balanced Mach 2 jet operating total temperatures (313–1370 K), the first-order helical mode $n = 1$ came closest to predicting the correct angle of emission to the shear layer. Moreover, this mode experienced the maximum total growth at Strouhal numbers below $St = 0.2$, which are representative of the peak noise emission. Axisymmetric structure is only important at high Strouhal numbers, and supersonic instability waves become more prominent at high jet temperatures, midrange frequencies $0.2 < St < 0.3$, and small angles to jet axis. The Kelvin-Helmholtz instability begins to diminish in importance with increasing jet temperatures.

Although the numerical results of this paper are important for understanding Mach wave generation, several important avenues need to be pursued to improve the overall value of the Rayleigh model. First, the assumption of equal initial amplitudes for each of the fundamental modes is arbitrary. A study of the jet shear layer receptivity at the nozzle exit to various modal disturbances needs to be undertaken, particularly for high-temperature jets. This could be done either experimentally or numerically.

The authors were surprised to see that the second-order helical mode $n = 2$ was extremely significant for Strouhal numbers above 0.15. The present analysis should be extended to include the third-order helical mode $n = 3$ to examine the importance of this mode. The Rayleigh model loses its significance if too many modes need to be included in the analysis. This is because the existence of many modes may suggest a nonlinear interaction process and cannot be modeled by the linear Rayleigh's equation. In such cases, the nonlinear interaction may be modeled using the parabolized stability equation. This approach has been successfully applied for boundary layers[13,14] and work is currently underway to develop models for jets. The results of this paper indicate that more modes are significant with increasing jet total temperature since they begin to attain supersonic phase speeds relative to the ambient sound speed. Without specific knowledge of the energy distribution between various modes in the initial shear layer, multiple important modes would reduce the utility of the Rayleigh model as the initial amplitude of these waves cannot be determined. As a consequence, dominant noise radiating modes cannot be identified, and devising schemes to control and minimize noise radiation becomes difficult.

Also still unresolved is the issue concerning the loss of supersonic phase speed predicted by the vortex sheet model after applying the viscous Orr-Sommerfield equation to achieve analytic continuation into the damped wave region for supersonic instability waves. This procedure is unnecessary for the Kelvin-Helmholtz instability waves.

The present analysis does indicate that the measured directivity of the principle Strouhal number component, as exemplified in Fig. 10, most likely represents some combination of fundamental modes. The Fig. 10 results suggest that experimental data must be relatively free of acoustic reflections, free from undesirable noise contamination, and have a high spatial resolution to conduct such a comparison.

Future work by the present authors using linear instability wave theory will also examine the possibility of selectively varying the mean velocity profile to diminish the growth rate of the first-order helical mode. This prototype effort is an attempt to study a means for suppressing the Mach wave emission mechanism. This work will also include application of an external low speed (i.e., $M_f \leq 0.3$) flowfield to account for forward flight effects.

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